# The Tully–Fisher Relation : A Numerical Simulator's Perspective

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**Abstract.** In this brief contribution, I will outline hopes of understanding the origin of galaxy scaling relations using numerical simulations as a tool to gain understanding. The case of the Tully–Fisher relation for disk galaxies is used as a working example to illustrate the modest achievements to date and the difficult tasks ahead.

## 1 Overview

Galaxies, like people, come in a variety of shapes and sizes. Like people, galaxies tend to have many features in common, but explaining in detail the origin of these features can be a difficult task. Understanding why the luminosity of a disk galaxy should scale as a power of its circular velocity is a bit like understanding the relation between a person's weight and his or her belt length. There are basic governing principles — bodies are (crudely speaking) similarly shaped bags of water and galaxies are (crudely speaking) similarly structured gravitational objects — but other factors creep in when you start giving it more careful thought. A skinny basketball player and a jockey might have the same waistline, but their body masses could differ by more than 50% because of their height difference. Why couldn't two galaxies lying in potential wells of the same circular speed differ in luminosity by, say, a factor 3 because of differences in their gas dynamic/stellar evolutionary histories? It appears nature does not allow this to happen; the scatter about the Tully–Fisher relation is remarkably small (see Giovanelli and others in this proceedings).

The remarkable nature of such a tight correlation in non–trivially linked physical properties is made apparent when one considers their complicated birthing process shown schematically in Figure 1. This picture was laid out theoretically in the late 1970's, and the seminal paper of White and Rees (1978) cemented the elements together within a modern, hierarchically clustering framework. In hierarchical models, gravity amplifies density perturbations on ever–increasing mass and length scales, driving an overdense, filamentary/knotty network which evolves in a nearly self–similar fashion. On mass scales roughly between $10^8$ and $10^{12}\mathrm{M}_\odot$ and in the absence of significant non–gravitational heat input, cooling via radiative processes removes thermal pressure support from the baryons. This process acts to concentrate the baryons within an assumed, dominant halo of

dark matter. Once self–gravitating, star formation is ignited in a manner poorly understood from first principles (hence the "black box" in the figure) and *poof!* we end up with a disk galaxy rotating in its dominant dark halo.

Given that rotation speed is a direct measure of total mass within a fixed density contrast $M_\delta$ (see below), then a suspiciously simple interpretation of the tight, observed Tully–Fisher relation in the context of Figure 1 is that cooling and star formation are highly regular and dependent primarily on $M_\delta$. Such a simple picture appears to be true for the structure of collisionless halos formed from hierarchical, gravitational clustering. A single characteristic function with parameters smoothly varying with mass appears to describe the density and velocity structure of collapsed objects (see White's contribution in this volume).

The situation in Figure 1 is simplistic in a number of ways. Formation of a single galaxy actually entails a network of such segments, inter-connected in a manner reflecting the particular merger history of that object. An ensemble of equal mass objects observed today will naturally arise from a variety of merger histories/inter-connections. Why doesn't this variety evidence itself as a large scatter in the Tully–Fisher relation? (Eisenstein in this volume presents a similar argument from a slightly different perspective.)

## 2 Looking Behind

Numerical simulations with gas dynamics are now beginning to be used to address the origin of disk galaxies and the Tully–Fisher relation (Katz & Gunn 1991; Evrard, Summers & Davis 1994, Navarro & White 1994, Steinmetz & Müller 1994; Navarro & Steinmetz 1996; Tissera, Lambas & Abadi 1996; Groom 1997). Most of these simulations ignore star formation altogether. Those in which it was included failed to form a disk of stars, forming spheroids instead. So the best we can do at the moment is analyze the gas disk properties. An idea of where we stand is shown in Figure 2, which compares the cold, gas mass in the galaxy to its circular speed. Data shown are from Navarro & White (1994; hereafter NW) and from a unpublished P3MSPH simulation by myself of a random, 16 Mpc ( $\mathrm{H_o} = 50$ km s$^{-1}$ Mpc$^{-1}$) patch of a standard cold dark matter universe. The characteristics of the simulation are identical to that detailed in Evrard *et al.* (1994), with the exception of it being a random, (instead of constrained cluster) spatial region and it being evolved to the present (instead of $z = 1$). The interested reader should consult these papers for further details. The "raw" points in the figure use the peak in the measured circular speed of the gas disk, while the "corrected" points enforce centrifugal equilibrium at that point in the rotation curve. The correction is necessary because the size of the disks is within a factor of a few of the spatial resolution limit of the simulation. The dotted line in the figures has slope 2.45.

The good news from Figure 2 is that the scatter in both data sets is quite small, in fact, smaller by a factor 2 than typical observed values. The bad news is that the left hand panel of the figure is a dishonest comparison of two independent experiments. The NW rotation speed is actually $\sqrt{GM/r}$ measured at a

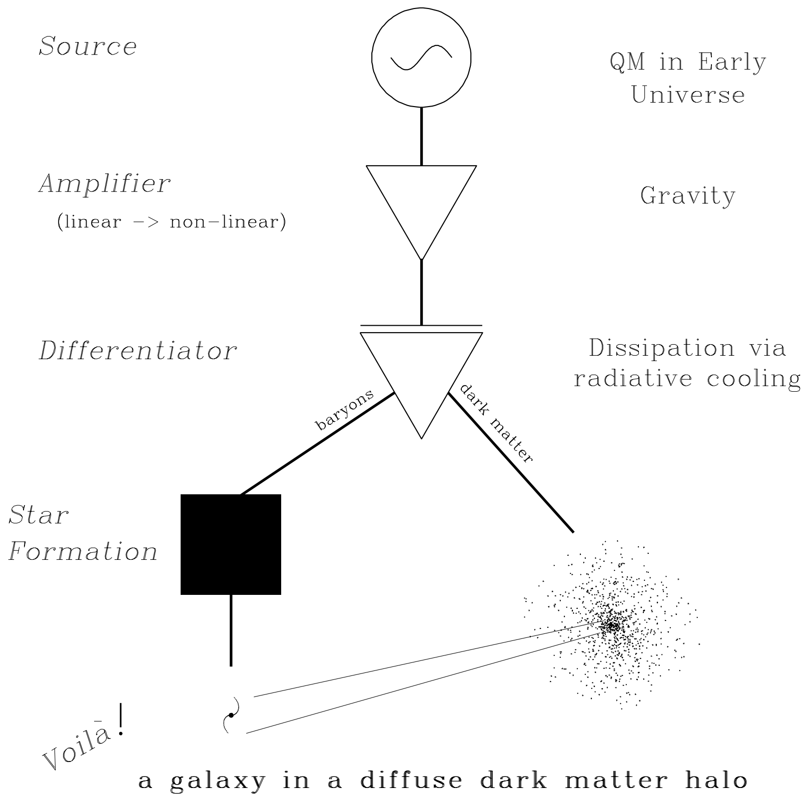


**Fig. 1.** Schematic illustrating the basic principles behind the White–Rees picture of galaxy formation discussed in the text.

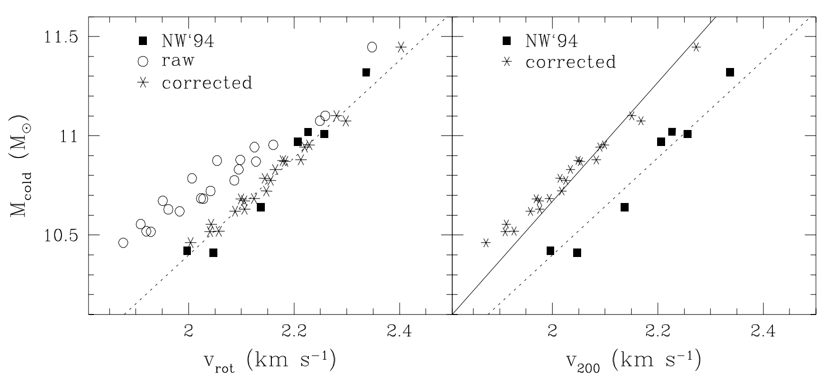


**Fig. 2.** Cold gas mass versus rotation speed derived from gas dynamic simulations of Navarro & White 1994 (NW) and Evrard (1995, unpublished). See text for a discussion.

density contrast of 200 with respect to the critical background. Going back and measuring the same quantity for the disk galaxies in the P3MSPH simulation (the "corrected" values in the right panel) results in a systematic offset in the intercept of the two data sets. This intercept is not due to different values of Hubble constant, both use $h = 0.5$, or cosmic baryon fraction, since both assume $\Omega_b = 0.1$ and $\Omega = 1$. In this cosmology, the radius and total mass defining a density contrast of 200 at the present epoch are

$$r_{200} = \sqrt{\frac{2}{\delta}}\,\frac{v_{200}}{\mathrm{H_o}} = 100\left(\frac{v_{200}}{100\ \mathrm{km\ s^{-1}}}\right) h^{-1}\ \mathrm{kpc} \tag{1}$$

$$M_{200} = 2.32 \times 10^{11}\left(\frac{v_{200}}{100\ \mathrm{km\ s^{-1}}}\right)^3 h^{-1}\mathrm{M}_\odot \tag{2}$$

If all the baryons within this density contrast cool and sink to the center of the halo, then the cold, galactic mass will simply be $\Omega_b M_{200}$. This implies a Tully–Fisher relation shown as the solid line in the right panel (using $h = 0.5$ as in the simulations). One interpretation of this panel is that the P3MSPH treatment is allowing nearly all the gas to cool in the halos while NW's treatment allows half the baryons to cool, with the remainder in a tenous, hot halo. It remains to be seen if this interpretation is correct but, at any rate, the offset between the two data sets is most likely numerical in origin, since both are attempting to model essentially identical physical situations. The silver lining here is that the small degree of scatter in the relation appears insensitive to the detailed numerical treatment.

## 3 Looking Ahead

The example above illustrates our current level of uncertainty in modeling just some of the physical processes associated with Figure 1. The black box of star formation is largely unexplored territory. Presumably different physical and numerical parameterizations for star formation and feedback will lead to an even larger range of possible answers than that illustrated in Figure 2.

On the bright side, a comparison between codes attempting to model the branch above the differentiator in Figure 1 (gravitational clustering without radiative cooling) indicate there is quite good agreement in the gas and dark matter solutions over the dynamic range presently accessed by such experiments, that is, density contrasts up to about $10^4$ (Frenk, White *et al.* 1997, in preparation). Similar comparisons including cooling will ultimately enable sorting out of physical versus numerical effects.

In the realm of galaxy scaling relations, theorists are in the typical position of attempting to understand current observations; predictive power is tenuous at best. From the excellent new data presented at this meeting, particularly in the area of evolution in the scaling relations at moderate to high redshift (*e.g.*, the contributions of Franx, Pahre, Schade, Guzman, Dickinson, and Ziegler in

this volume), it seems the observers are accelerating their pace! Modeling this wealth of data presents a formidable challenge for the foreseeable future.

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